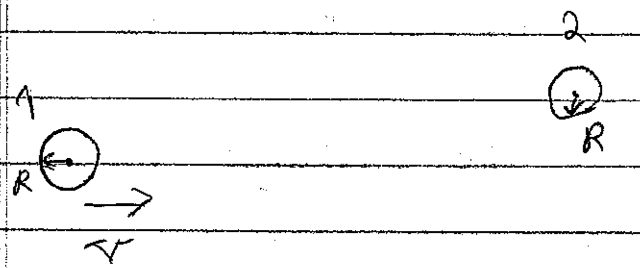
Lec 12: 2/25/2010

Freeze-out of Weak Interactions:

For a particle to be in thermal equilibrium, it must have efficient interactions with the rest of the particles in the universe. Recall that the time average of the energy of a particle that is in thermal equilibrium is equal to thermal average of energy. Therefore, if the particle starts with an arbitrary energy, it needs interactions in order for its energy to change. Thes interactions guarantee the exchange of energy between the particle and the surrounding (i.e. other particles. In a static universe, interactions between particles become eventually efficient as one can wait for an arbitrary long time. In an expanding unive however, interactions must happen at a rate that is faster than, or comparable to, the Hubble expansion rate. Otherwise interactions cannot catch up with expansion and maintain thermal equilibrium at all times.

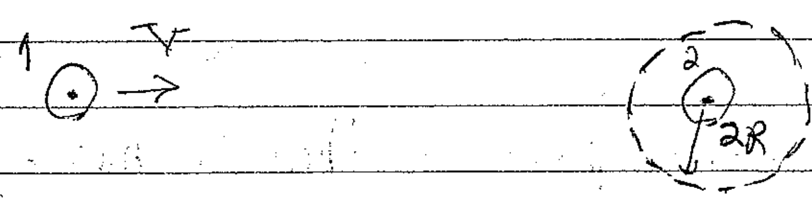
To elaborate on the rate for interaction between particles, lets consider the analogy with colliding billiard balls. Consider ball 1 that is shot toward ball 2 that sits at rest:



Initially ball 1 has a velocity in the horizontal direction. If it does not hit ball 2, then it will continue moving in the horizontal direction. If it hits ball 2, then it will scatter at an angle

$\theta \neq 0$. We can find out about a collision by looking at all angles to see ball 1. If ball 1 is not found anywhere except in the forward direction $\theta \approx 0$, then no collision has happened.

In order for a collision to happen, the distance between the center of ball 1 must come within a distance $\leq 2R$ from the center of ball 2. This defines a cross-sectional area $\pi(2R)^2$ around the center of ball 2. If ball 1 enters this area, then collision happens. Otherwise, there will be no collision between the balls.



The cross section for collision can be measured if we keep shooting balls at ball 2 and then count the number of balls scattered at angle $\theta \neq 0$. The cross section is defined as:

$$\sigma \equiv \frac{N_{out}}{N_{in}}$$

Here N_{out} is the number of balls scattered at $\theta \neq 0$ per unit time, while N_{in} is the number of incoming balls per unit area per unit time

In the case of particles we have "particles" instead of balls and "interaction" instead of collision. The difference is that the scattering cross section σ has no geometric meaning in this case, and must be calculated from microphysics.

Now consider a particle that is moving through the surrounding (other particles). The interaction rate of that particle is defined as the number of scattering that it will undergo per unit time.

The interaction rate Γ depends on the scattering cross section (σ), the number density of particles that surround it (n), and the relative velocity between the interacting particles (v_{rel}).

It is easy to verify that the interaction rate between two particles is:

$$\Gamma = n \sigma v_{rel}$$

In general, since particles in the surrounding have different energies and velocities, we need to take the average:

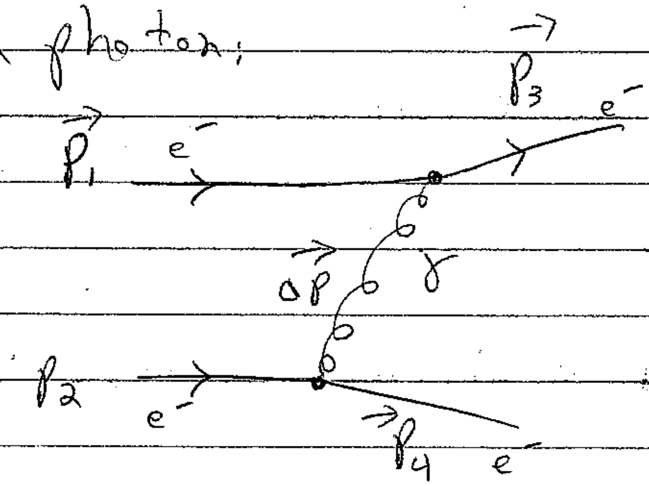
$$\Gamma = n \langle \sigma v_{rel} \rangle$$

In the case of a thermal bath, $\langle \rangle$ denotes the thermal average.

We are now ready to estimate the rates for electromagnetic and weak interactions in the early universe:

– Electromagnetic interactions. Consider two electrons in the relativistic regime with energies E_1 and E_2. It is usually easier to go to the center-of-mass frame where $\vec{p}_1 + \vec{p}_2 = 0$. In this frame the electrons have the same energy $E \sim \sqrt{E_1 E_2}$.

The electrons interact by the exchange of a photon:



The exchanged momentum $\Delta\vec{p} = \vec{p}_3 - \vec{p}_1 = \vec{p}_2 - \vec{p}_4$ is carried by the photon. Since photon is massless, then momentum $\vec{\Delta p}$ amounts to an energy $\Delta E = |\Delta \vec{p}\,|$ for the photon. However, it is easy to see that $|\vec{p_4}| = |\vec{p_3}| = |\vec{p_2}| = |\vec{p_1}| = E$. This implies that the photon carries zero energy. It amounts to an uncertainty $|\Delta \vec{p}\,|$ in the energy of the exchanged photon, which can exist only for a short period of time $|\Delta \vec{p}\,| \sim \Delta t^{-1}$ according to the Heisenberg uncertainty principle (we use natural units where $\hbar = c = 1$).

Since photon is massless, it moves at the speed of light. Therefore the exchanged photon exists within a distance range $\Delta r \sim \Delta t \sim |\Delta \vec{p}\,|^{-1}$.

Note that $|\Delta \vec{p}\,| \leq 2E$, and hence:

Δr ...

The scattering cross section will then be:

$$\sigma \sim \Delta r^2 \sim \frac{1}{E_1 E_2}$$

In thermal equilibrium, we have to average over σv_{rel} ($v_{rel} \sim 1$ since electrons are relativistic):

$$\langle \sigma v_{rel} \rangle \sim \frac{1}{T^2}$$ (thermal average of energy is $\sim T$)

To be complete, we should notice that electrons exchange photon because of their electric charge. The emission and absorption of the exchanged photon would not happen if the electric charge of the electron was zero. We therefore expect that:

$$\Delta r \sim \frac{e^2}{\sqrt{E_1 E_2}} \Rightarrow \sigma \sim \frac{e^4}{E_1 E_2} \Rightarrow \boxed{\langle \sigma v_{rel} \rangle \sim \frac{\alpha}{T^2}}$$

Here $\alpha \simeq \frac{1}{137}$ is the fine structure constant.

We can now find Γ_{em}:

$$\Gamma_{em} = n \langle \sigma v_{rel} \rangle$$

In thermal equilibrium $n \sim T^3$, and hence:

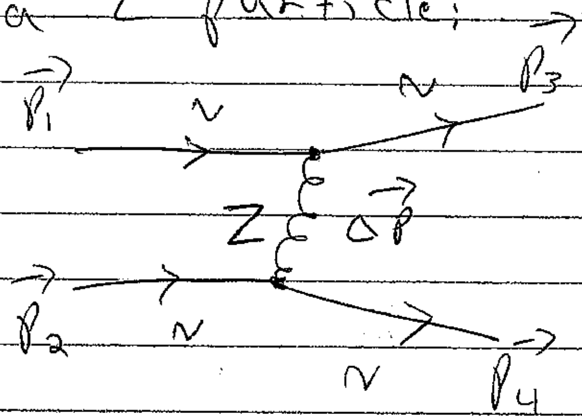
$$\boxed{\Gamma_{em} \sim \alpha T}$$

To compare Γ_{em} with the expansion rate $H \sim \frac{T^2}{M_P}$, we find that:

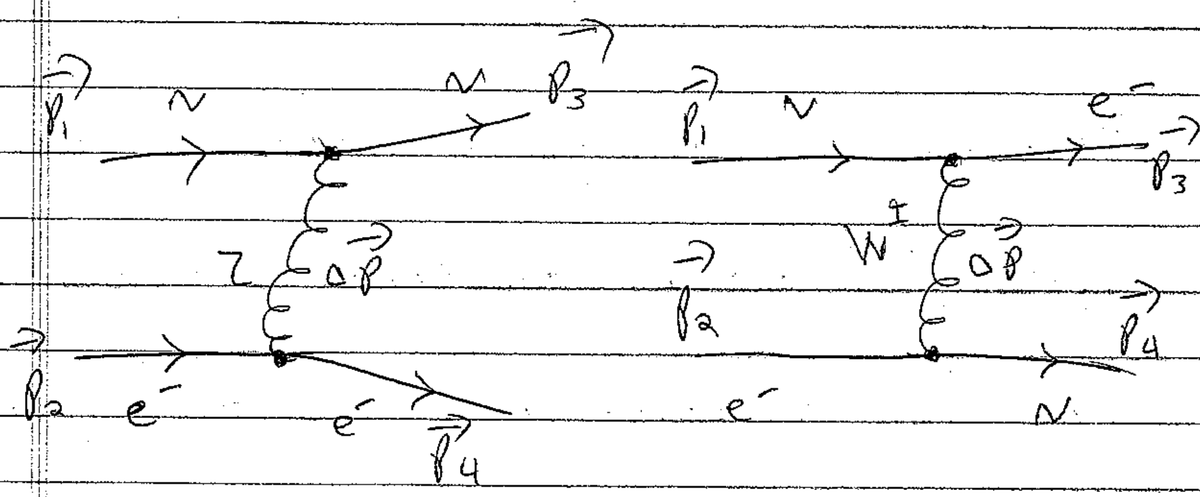
$$\boxed{\Gamma_{em} > H \Rightarrow \alpha^2 T > \frac{T^2}{M_P} \Rightarrow T < \alpha^2 M_P \sim 10^{15}\,\text{GeV}}$$

Therefore electromagnetic interactions are indeed very efficient at temperatures below 10^{15} GeV. They can estabish and maintain thermal equilibrium between charged particles.

- Weak interactions: As we discussed in the previous lecture neutrinos have only weak interactions because they are electrically neutral. Two neutrinos interact by exchanging a Z particle;



A neutrino interacts with an electron by exchanging Z and $W^{\pm}$ particles,



Here we assume that the energy of interacting particles is $\ll m_Z, m_{W^{\pm}}$, but much larger than their mass (so that neutrino and electron are in the relativistic regime).

By going to the center-of-mass frame, we again find that $|\Delta \vec{p}| \leq 2\sqrt{E_1 E_2}$. Since Z and $W^{\pm}$ are massive, this amounts to an uncertainty $\Delta E \sim \sqrt{m^2_{Z,W^\pm} + E_1 E_2} \simeq m_{Z,W^\pm}$ in the energy of exchanged $Z, W^\pm$. This can exist only for a short period of time $\Delta t \sim m^{-1}_{Z,W^\pm}$. During this time the $Z, W^\pm$ can travel a distance $\frac{|\Delta \vec{p}|}{m_{Z,W^\pm}} \times \frac{1}{m_{Z,W^\pm}}$ (note that the exchanged $Z, W^\pm$ are non-relativistic, hence $v = \frac{|\vec{p}|}{m_{Z,W^\pm}}$).

We therefore find that:

$$\Delta r \sim \frac{\sqrt{E_1 E_2}}{m^2_{Z,W^\pm}} \Rightarrow \sigma \sim \Delta r^2 \sim \frac{E_1 E_2}{m^4_{Z,W^\pm}}$$

Repeating what we did for electromagnetic and taking the thermal average), we find:

$$\boxed{\Gamma_{weak} \sim G_F^2 T^5}$$

Here $G_F \cong 1.17 \times 10^{-5}$ GeV^{-2} is the Fermi Constant.

Comparing Γ_{weak} and H, we see that:

$$\Gamma_{weak} > H \Rightarrow G_F^2 T^5 > \frac{T^2}{M_P} \Rightarrow T^3 > \frac{1}{G_F^2 M_P}$$

$$\Rightarrow \boxed{T > 1 \text{ MeV}}$$

The important result is that weak interactions are efficient if $T > 1$ MeV. They freeze-out and drop out of equilibrium at $T < 1$ MeV (or $t > 1$ sec). As we will see next, this is a very important moment in the history of the universe. It is the onset of big bang nucleosynthesis.